

COMPUTER MODELING OF THE ACCELERATION PROCESS IN A HEAVY-CURRENT LINEAR ION ACCELERATOR

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1. Reference [1] describes calculations of charge motion in an electromagnetic field, allowing for interaction. The basic parts of the calculation are: numerical integration of the equations of particle motion, and numerical solution of the boundary problems for equations of the elliptic type. The same reference makes a comparative analysis of schemes suitable for solution of a difference analog of the Dirichlet problem for a rectangular region in a cylindrical system of coordinates, and one economical scheme is described. The present paper applies the method examined in [1] to computer modeling of the acceleration process in a heavy-current linear accelerator. The approximate model of the physical process is based on the method of "large molecules." It involves individual description of the motion of "large particles" (packets of particles) in their own field (which is calculated numerically after appropriate averaging of the density) and in external fields (electric and magnetic), calculated with a difference analog of the boundary problem for the Poisson (Laplace) equation in cylindrical geometry.

The advantage of this kind of examination lies in the fact that the averaged quantities are not differentiated, but integrated, which considerably increases the accuracy of calculation when the charges are grouped in a bunch and the charge density varies rapidly in space. A shortcoming of this kind of model is the transition to the solution of a number of equations-limited by the capabilities of the computer-describing the motion of grouped charges. The choice of the number of such charges, modeling the actual process, is made from numerical experiments.

Similar models have been used earlier in problems of rarefied plasma [2-4], in calculating the influence of a space charge in an accelerator (reference [5]), and in investigations of klystron bunching of a heavy-current beam with strong velocity modulation of the particles [7].

2. We consider the model of the accelerator to have the form of a continuous cylinder of radius R and length L (in the cylindrical coordinate system $0 \leq r \leq R$, $0 \leq z \leq L$, $0 \leq \varphi \leq 2\pi$). The case examined is a quasi-electrostatic field. Under the assumption of axial symmetry, the potential of the external electric field is determined from the equation

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0. \tag{2.1}$$

Here the potential on the side surface of the cylinder is given by

$$\begin{aligned} u(r, 0, t) &= 0, \quad \text{or} \quad \partial u / \partial z_{z=0} = 0, \\ u(R, z, t) &= f_1(z) \cos \omega t + f_2(z) \cos(\omega t + \varphi), \\ u(r, L, t) &= 0, \quad \text{or} \quad \partial u / \partial z_{z=L} = 0. \end{aligned} \tag{2.2}$$

By introducing $f_1(z)$ and $f_2(z)$, we make it possible to assign potentials at two resonators with a phase shift φ . In the subject problems the functions $f_1(z)$ and $f_2(z)$ were given as being linear in the gap, as was assumed for thick-walled drift tubes. It should be noted, however, that to carry out the calculations, we must assume values, on the surface $r = R$, for any of

the mesh functions $f_1(z)$ and $f_2(z)$ determined at a series of points in the interval $0 \leq z \leq L$. Thus, for complex gaps, the potential at the boundary can be removed from an electrolytic bath. We shall assume

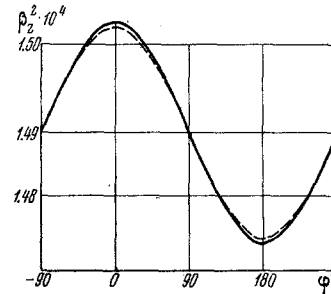


Fig. 1

that within the cylinder $f_1(z)$ and $f_2(z)$, respectively, generate potentials $\Phi_1(r, z)$ and $\Phi_2(r, z)$, which satisfy Eq. (2.1) and are associated with potentials of the left ($\Phi_1 \cos \omega t$) and right ($\Phi_2 \cos(\omega t + \varphi)$) resonators. Their total potential Φ is defined as

$$\Phi = \Phi_1 \cos \omega t + \Phi_2 \cos(\omega t + \varphi).$$

It is assumed that the cylinder is located in a steady magnetic field such that $H_\varphi = 0$, where H_φ is a component of the magnetic field-intensity vector. We neglect the high-frequency magnetic field in the acceleration of nonrelativistic ions.

The external magnetic field is determined from the same equation (2.1) as $u = H_z$ under the conditions

$$\begin{aligned} \frac{\partial}{\partial z} H_z(r, z) \Big|_{z=0} = 0, \quad \frac{\partial}{\partial z} H_z(r, z) \Big|_{z=L} = 0, \\ H_z(R, z) = f_3(z). \end{aligned} \tag{2.3}$$

It is assumed also that at time $t = t_0$ there are no charges inside the cylinder. Then, beginning at $t = t_0$, we assume that a cylindrical uniformly charged beam is applied with a given current to the accelerator input. This current is carried into the accelerator by "large particles," each of which represents n elementary particles with charge e and mass m , united into a single two-dimensional particle, a circle whose center is on the cylinder axis. A charge $q = ne$ is uniformly distributed over the circle. At time $t = t_0$ this particle is described by the quantities $r_{0k}, z_{0k}, \dot{r}_{0k}, \dot{z}_{0k}$ ($k = 1, 2, \dots, N$).

Let us dwell on the determination of these quantities. Let the velocity of the particles injected into the accelerator be known and equal to β_z . Having selected a rather small time increment Δt , we determine the charge passing through the left end of the cylin-

der. Let this charge be Q . Geometrically it may be represented as a cylinder of radius R_0 (the radius of the beam), with height $\beta_z \Delta t$. We divide this cylinder into N toroidal volumes, considering that each of the volumes is generated by rotation of the rectangle with its base parallel to the axis of the cylinder and equal to $\beta_z \Delta t$, and by the side Δr_k , rings ($k = 1, 2, \dots, N$) of equal area being described during rotation of the lateral sides. It is clear that this volume contains a charge $q = Q/N$.

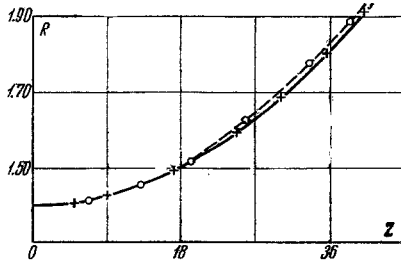


Fig. 2

The center lines of these toroidal volumes were assumed to be "large particle." Therefore,

$$r_{0k} = R_0 \sqrt{(2k-1)/2N}, \quad z_{0k} = \beta_z z.$$

As regards z_{0k} and \dot{r}_{0k} , we must put

$$z_{0k} = \frac{1}{2} \beta_z \Delta t, \quad \dot{r}_{0k} = 0.$$

However, it can be assumed that $r_{0k} \neq 0$. This enables us to inject into the accelerator a convergent or a divergent beam, and also to simulate the scatter in the transverse thermal velocities of the beam particles. If it is assumed that, at the moment of injection, the center of particle gravity is located at the section $z = 0$, we have $z_{0k} = 0$.

Moving under the influence of the external (electric and magnetic) fields, the introduced N particles will occupy a new position in the cylinder at the time $t_0 + \Delta t$. We note that the potential of their space charge at this time satisfies the equation

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{4\pi\rho}{\epsilon}. \quad (2.4)$$

Here ρ is the space charge density, and ϵ is the dielectric constant of the medium. It follows from the formulation of the problem that at the left end of the cylinder, the condition

$$\partial \psi / \partial z |_{z=0} = 0 \quad (2.5)$$

is satisfied.

Since the lateral surface (Σ) of the cylinder is considered to be conductive

$$\psi |_{\Sigma} = 0. \quad (2.6)$$

The use of condition (2.6) in the gaps is valid only when the size of the gap is small in comparison with the remaining region; therefore, we can assure, without great error, that on the whole lateral surface of

the region that we are considering, the potential of the space charges is equal to 0.

As regards the right end, we set up the so-called mirror reflectance condition for the beam

$$\partial \psi / \partial z |_{z=L} = 0. \quad (2.7)$$

Condition (2.7) does not follow from the formulation of the problem, but it exerts some slight influence on the motion of the charges, even at some distance from the right-hand end, to the left of the section $z = z_0$, which was chosen from numerical experiments. The choice of z_0 can also be made from the consideration that in a metal tube two bunches separated one from the other by a distance large in comparison with the tube radius virtually do not interact.

It is considered that only in the region $0 \leq z \leq z_0$ will the motion of the grouped charges model the actual process.

Solving the problem (2.4)–(2.7) for the time $t_0 + \Delta t$ and repeating the start-up process for the N particles with the above coordinates, we find it necessary now to determine the location of $2N$ particles (if none escaped through the lateral surface or the end), moving under the influence of both the external field and the self field, and then we come to the solution of this problem at time $t_0 + 2\Delta t$. In a similar way, we determine the position within the cylinder of the bunched charges, with the following variables: radius r and coordinate z , velocity with respect to r and velocity with respect to z , and the constant value of q at time $t_0 + 3\Delta t, \dots, t_0 + k\Delta t, \dots$

The motion of these charges (see [1]) is described by the relations

$$r'' = -\frac{\partial \kappa}{\partial r}, \quad z'' = -\frac{\partial \kappa}{\partial z},$$

$$\kappa = \Phi + \Psi + \frac{1}{2} A_{\phi}^2, \quad H_z = \frac{1}{r} \frac{\partial}{\partial r} r A_{\phi}. \quad (2.8)$$

Because of the time function in the form (2.2), we should anticipate in the calculation, and this is actually observed, that in the region $0 \leq z \leq L$ there will be set up a regime having a period coincident with the period of change in the external electric field. Provision was made for the storage of the steady-state regime at the section $z = z_0$. At each step of the period, the values of the potential at the nodes of the grid of the straight line $z = z_0$, the coordinates of the particles located at the section $z = z_0$, or of these which have just passed this section, and their velocities are recorded in the magnetic memory. These records could be used as initial conditions for the next part of the problem, when a periodic regime is assumed at the left end of the cylinder. It thus becomes possible to make a detailed calculation and to enlarge considerably the possibility of modeling processes in an accelerator with a large number of resonators.

3. The method of integration for the equations of motion and for the numerical solution of the boundary problems has been rather fully described in [1]. In calculating the motion of the charges in the accelerator, it should be taken into account that the region where

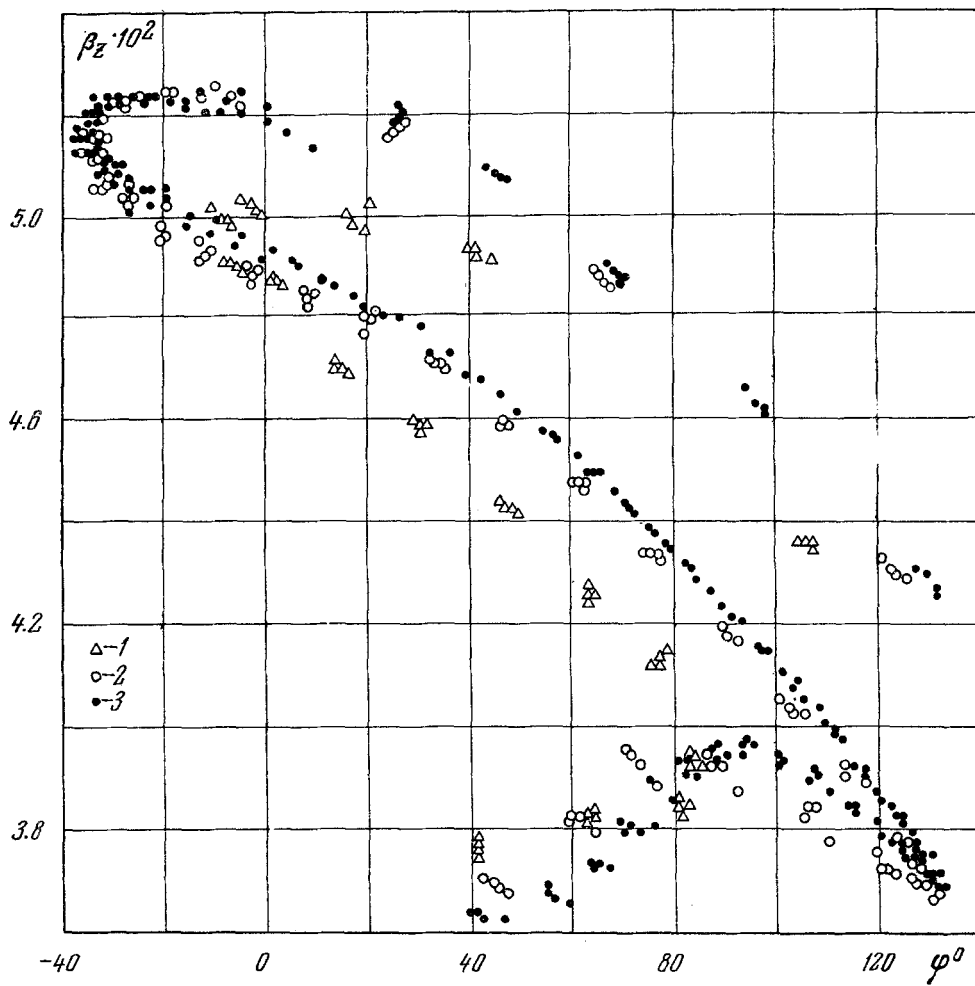


Fig. 3

the motion is examined is strongly elongated in the z direction ($R \ll L$). In this connection, in numerical determination of the potentials at the grid nodes, the author used the "unidirectional pivot method" that is most economical here. We shall describe the essentials of the method, using a Cartesian coordinate system.

Let us examine the system of equations

$$(\Lambda_1 + \Lambda_2)u = 0, \quad u|_{\Gamma} = \varphi, \quad (3.1)$$

$$\Lambda_1 u = \frac{u_{i+1j} - 2u_{ij} + u_{i-1j}}{h^2},$$

$$\Lambda_2 u = \frac{u_{i+1} - 2u_{ij} + u_{i-1}}{l^2}, \quad u(ih, jl) = u_{ij}. \quad (3.2)$$

Here Γ is the contour of the rectangular region ($0 \leq x \leq a$, $0 \leq y \leq b$); h and l are the steps of the rectangular grid; $i = 1, 2, \dots, N_0 - 1$; $j = 1, 2, \dots, M - 1$; $N_0 h = a$, $M l = b$.

To solve system (3.1) we can use the two iteration schemes

$$(E - 1/2 \Lambda_1) u^{n+1} = 1/2 (u_{ij+1}^n + u_{ij-1}^n), \quad u^n|_{\Gamma} = \varphi, \quad (3.3)$$

$$(E - 1/2 \Lambda_2) u^{n+1} = 1/2 (u_{i+1j}^n + u_{i-1j}^n), \quad u^n|_{\Gamma} = \varphi. \quad (3.4)$$

Here n is the number of iterations, and u^0 is any zeroth approximation to the solution of the boundary problem (3.1).

Schemes (3.3) and (3.4) with $h = l$ were examined in [7] as examples of block iterations.

j	$u^{(1)}$	$u^{(2)}$	$u^{(3)}$
0	0	0	0
1	0.095583	0.095766	0.095765
2	0.188813	0.189175	0.189174
3	0.277394	0.277926	0.277925
4	0.359144	0.359833	0.359832
5	0.432051	0.432880	0.432878
6	0.494319	0.495268	0.495266
7	0.544416	0.545461	0.545459
8	0.581107	0.582222	0.582220
9	0.603490	0.604648	0.604646
10	0.611012	0.612185	0.612183

It can be shown that processes (3.3)-(3.4) converge in the metric L_2 to the solution of (3.1) for any u^0 and any h and l .

In fact, because of linearity, for an error $G^n = u - u^n$ for schemes (3.3) and (3.4), respectively, we obtain

$$(E - 1/2 \Lambda_1) G^{n+1} = 1/2 (G_{ij+1}^n + G_{ij-1}^n),$$

$$(E - 1/2 \Lambda_2) G^{n+1} = 1/2 (G_{i+1j}^n + G_{i-1j}^n). \quad (3.5)$$

Expanding G^n in eigenfunctions, we have

$$G^n = \sum_{kp} a_{kp} q^{kp}, \quad q_{ij}^{kp} = \sin k i \alpha \sin p j \beta, \quad \left(\alpha = \frac{\pi h}{a}, \quad \beta = \frac{\pi l}{b} \right)$$

$$(k = 1, \dots, N_0 - 1; \quad p = 1, \dots, M - 1). \quad (3.6)$$

According to (3.5) and (3.6), we obtain

$$G^{n+1} = \sum_{kp} \rho_{kp}^{n+1} a_{kp} q^{kp}. \quad (3.7)$$

Here ρ_{kp} is determined for (3.3) and (3.4), respectively, as follows:

$$\rho_{kp}^{n+1} = \left(\frac{\cos p \beta}{1 + 2/l^2 h^2 \sin^2 1/2 k \alpha} \right)^{n+1},$$

$$\rho_{kp}^{n+1} = \left(\frac{\cos k \alpha}{1 + 2/h^2 l^2 \sin^2 1/2 p \beta} \right)^{n+1}. \quad (3.8)$$

It follows from (3.7) and (3.8) that schemes (3.3) and (3.4) converge. Now passing to the limit in (3.3) and (3.4), we have $\lim_{n \rightarrow \infty} u^n = u$ for $n \rightarrow \infty$, where u is the solution of system (3.1), a difference analog of the Dirichlet problem.

The quantities in (3.8) characterize the rate of extinction of the original error G^n . Thus, for scheme (3.3), from (3.7) and (3.8), for sufficiently small α , we have

$$\|G^n\| \leq (1 + 1/2 \pi^2 l^2 / a^2)^{-n} \|G^0\|. \quad (3.9)$$

However, in the case of scheme (3.4), in a similar way, for sufficiently small β , we obtain

$$\|G^n\| \leq (1 + 1/2 \pi^2 h^2 / b^2)^{-n} \|G^0\|. \quad (3.10)$$

On the basis of (3.9) and (3.10), we state that if $l/a > h/b$, the most economical scheme will be (3.3), and if $l/a > h/b$, then preference must be given to (3.4). Thus, when $h = l$ and $a > b$, we must use vertical pivot steps. A coarser grid in the longitudinal direction (for $a > b$) will only improve the convergence of scheme (3.4).

The conclusions regarding schemes (3.3) and (3.4) extend also to the case of a cylindrical coordinate system, when

$$-1/2 h \leq r \leq R, \quad 0 \leq z \leq L, \quad r_i = (i - 1/2) h, \quad z_j = j l,$$

$$\Lambda_1 u = h^{-2} \left(\frac{2i}{2i-1} u_{i+1j} - 2u_{ij} + \frac{2i-2}{2i-1} u_{i-1j} \right),$$

$$\Lambda_2 u = l^{-2} (u_{ij+1} - 2u_{ij} + u_{ij-1}).$$

For $l \geq h$ and $R \ll L$, in this case it is most expedient to use the scheme

$$(E - 1/2 \Lambda_1) u = 1/2 (u_{ij+1} + u_{ij-1}) - 1/2 l^2 f, \quad (3.11)$$

which converges to the solution of the ordinary five-point approximation equation

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = f. \quad (3.12)$$

As regards accuracy of calculation of the potentials at the grid nodes from scheme (3.11), it is quite satisfactory. We illustrate this in the following problem: find the solution of system (3.11) under the conditions

$$-0.475 \leq r \leq 0.475, \quad 0 \leq z \leq 1,$$

$$u(-0.475, z) = u(0.475, z) = \sin \pi z,$$

$$u(r, 0) = u(r, 1) = 0, \quad h = l = 0.05, \quad f = 0. \quad (3.13)$$

The table gives values of $u^{(2)}$ -solutions of the problem (3.11), (3.13) at the nodes $z_j = j l$ ($j = 0, 1, \dots, M - 1$) of the straight line $r = 0.025$. For comparison, we present values of $u^{(3)}$ obtained by the well-known longitudinal-transverse pivot method at the same nodes, as well as the values of $u^{(1)}$ -solutions of the differential problem (3.12), (3.13), which has the form

$$u = I_0(\pi r) \sin \pi / 0.475 z.$$

Here I_0 is a Bessel function of imaginary argument.

The numerical solution was obtained by carrying out iteration for

$$\max_{ij} |u_{ij}^{n+1} - u_{ij}^n| < 10^{-6}.$$

4. The basic justification for numerical methods of calculating external electromagnetic fields, fields of a space charge, and for methods of integrating equations of motion has been given in [1]. Earlier we demonstrated the possibility of using scheme (3.11) to calculate potentials when $R \ll L$. However, in practice, the accuracy of the resulting solutions depends on the calculation parameters which, moreover, determine the machine time expended in the calculation. It is clear that the greater the number of nodes taken in the region of calculation, and the smaller the consolidation of the charges, the more accurate the results, and the closer the modeling to the actual process. But both the number of nodes and the number of particles in the experiment are still dictated by the capabilities of the computer.

Typical conditions for calculation of specific problems, and an experimental check on the accuracy in modeling the basic processes are described below.

The fields were usually calculated in the region $(-h/2 \leq r \leq R, 0 \leq z \leq L)$ at node 781 (11×71), including the nodes at the boundaries. The ratio of L to R usually went up to 40; the ratio of l to h reached a value of six here. With these parameters about 20 sec was required for calculating the field on a commercially available machine when determining the potential of a space charge, when the zeroth approximation was assumed to be the values of ψ , obtained in the previous step in time, and about a minute, if zero was taken as the zeroth approximation. Calculation of the self field constituted the main part of the time expended on the calculation. As far as the accuracy of the obtained solution is concerned, it was quite satisfactory. The latter was verified in analytical solutions for the schemes described in [1]. On introduction of scheme (3.11) into the calculation, a comparison was made with solved problems. Results of the comparison were similar to those presented in the table.

The field determined only at node 781, on integration of the equations of motion, was interpolated linearly at the point having the coordinates of the calculated bunched particle.

In the majority of specific variants, at least two or three nodes of the grid are placed along the z axis over the length of the accelerating gap. In some cases an even more detailed grid was required. Here the region for the calculation of the particle motion ($0 \leq z \leq L$), contained only from three to eight accelerating periods. If this was not enough, the modeling of the process throughout the entire accelerator was reduced to the solution of two or more problems. On solution of the first of these at the section $z = z_0$ (usually in the middle of the drift tube) during one high-frequency period the steady-state process was stored in the magnetic memory: the coordinates and velocities of the particles, and also the values of the space charge fields at the node, $z = z_0$. These data were used as initial values at the left end in calculating the next section of the accelerator. Repetition of similar procedures permitted us to calculate the dynamics of the particles in an accelerator of any length. The time interval was usually defined as $\Delta\tau = T/36$, where T is the period of the external high-frequency electric field.

The calculation accuracy for the external field and for the dynamics of the particles, without allowing for the space charge, is illustrated in Fig. 1 which shows the increment in particle energy as a function of the phase of transit over the gap center. The initial energy of the protons is $W_0 = 70$ keV, the voltage at the gap is $U = 2.56$ keV. For comparison, the dashed line shows the analytical dependence [8]. The deviation from this relation does not exceed 1% for all the points.

Figure 2 shows the dependence $R = R(z)$, obtained in numerical solution of the problem of the spreading of a uniformly charged cylindrical beam under the influence of a space charge in a space free of external fields, the phase volume of the beam being $V = 0$, the initial beam radius $R_0 = 1.41$ cm, the beam current $I = 1$ amp, the particle energy $W_0 = 700$ keV, the time interval $\Delta\tau = (400/36)$ C (C is the speed of light in vacuum).

At each time interval four grouped particles with charge $q = (I/4)\Delta t$ are introduced across the section $z = 0$.

The dashed line gives the analytical relation $R = f(z)$ for a beam with the appropriate parameters ([9, 10]). Rather good agreement is obtained even for considerable divergence of the beam. The calculated field at the beam surface close to the injection section coincides, with high accuracy, to the field of an infinite, uniformly charged cylinder of corresponding radius.

The question of the accuracy of calculating the space-charge fields with the particles bunched in a cluster in a linear accelerator was investigated separately.

The minimum number of particles, the maximum time interval, and the maximum length of the calculation mesh along the z axis must be determined for each class of problems on the basis of the convergence of the solutions for successive decrease in the above parameters. The class of problem is characterized by the order of magnitude of the voltage at the gap, the wavelength of the external elec-

tric field, the order of magnitude of the beam current strength, the energy of the particle, the magnitude of the external magnetic field, and the relationship between the longitudinal and transverse dimensions of the calculation region. Thus, for beams with a current strength of 0.5 to 3 amp and energy of 0.5 to 10 Mev, moving in an external magnetic field of up to 10 000 Oe, with a voltage at the gaps of up to thousands of kV, when the length of the calculation region exceeded the cylinder radius by a factor of almost 40, injection of four particles for each 10° was quite valid. It made sense to calculate the fields in this case using 11×71 nodes. In confirmation of this, Fig. 3 shows the distribution of particles in the phase plane of longitudinal motion ($\beta = \varphi$) for the following combinations of intervals of time, space, and number of particles determining the accuracy of field calculations: 1 ($2l, 2\Delta\tau, N = 72$), 2 ($l, \Delta\tau, N = 144$), 3 ($0.5l, 0.5\Delta\tau, N = 288$). Here a beam was injected into an accelerator with an aperture radius $R = 2$ cm, the beam parameters being: initial radius $R_0 = 1.5$ cm, $I = 3$ amp, and injection energy $W_0 = 0.7$ MeV. The wavelength of the external electric field was $\lambda = 400$ cm, the synchronous phase was $\varphi = -30^\circ$. The length h of the mesh along the r axis was chosen to be $2/9.5$ cm. The intensity of the magnetic field at the boundary was $H_z(R, z) = 13 000$ Oe.

In condition (2.2), we assumed $f_2 = 0$. The function $f_1(R, z)$ varied linearly between the values: (0, 0), (6.07, 0), (10.11, 202), (22.78, 202), (27.18, 422), (40.94, 422), (40.94, 422), (45.71, 660.5), (60.58, 660.5), (65.73, 918), (81.73, 918), (87.29, 1194.5), (104.41, 1194.5), (110.32, 1490), (128.63, 1490), (134.93, 1805), (280, 1805). The first number in the parentheses is the z coordinate, and the second number is the corresponding value of $f_1(R, z)$. A number of variants were calculated with the intervals of time, distance, and the number of particles injected in one high-frequency period assuming various values. The results shown in Fig. 3 correspond to the steady-state regime.

The admission of 150–200 particles in one high-frequency period was used in a problem with continuous beam injection. The calculation was continued in this case until the instant of entry into a steady regime which was determined mainly by the particles not involved in the accelerator regime. However, in the majority of actual installations, such particles, in the course of several periods of acceleration, can either lose their transverse stability and settle out to the walls of the drift tubes (for example, in accelerators with hard focusing), or are filtered by artificial methods. In such equipment the beam is an array of clusters which are rather widely spaced, and do not interact. In modeling this kind of beam, it is sufficient to effect injection during only a single high-frequency period, and then to use all of the particles determined by the program memory (in our case, 2040). In this way it is possible to effect rather good modeling of some complex physical processes in the beam (thermal scatter of particle velocity, etc.).

The conducted investigations of this program indicate that it can be used for calculation experiments.

With reference to the matter of acceleration, the program can be very useful, not only in the design of specific equipment (the buncher, the initial part of the heavy-current accelerator, and the klystron tube), but also for improvement of some of the hypotheses of accelerator theory, primarily associated with a space charge, and the need to depart from a one-frequency theory.

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REFERENCES

1. V. A. Enal'skii, "The motion of particles in an electromagnetic field," Tr. Matem. in-ta AN SSSR, vol. 74, 1966.

2. O. Buneman, "Dissipation of currents in ionized media," *Phys. Rev.*, vol. 115, no. 3, p. 503, 1959.
3. J. Dawson, "Investigation of the doublestream instability," *Internat. atomic energy agency. Salzburg*, 1961.
4. V. A. Enal'ski and V. S. Imshennik, "A non-linear problem relating to the collision of clouds of rarefied plasma," *PMTF [Journal Of Applied Mechanics And Technical Physics]*, no. 1, 1965.
5. S. P. Lomnev, "Calculation of the phase trajectories of charged particles, involving Coulomb interaction in the buncher of a linear electron accelerator," *Dokl. AN SSSR*, vol. 135, no. 4, 1960.
6. S. E. Welber, "Ballistic analysis of a two-cavity finite beam klystron," *Transaction Inst. Radio Engineering N.Y.E.D.*, 598, 1958.

7. V. K. Saul'ev, *Integration of Parabolic Type Equations by the Mesh Method [in Russian]*, Fizmatgiz, 1960.

8. D. V. Karetnikov, I. N. Slivkov, V. A. Teplyakov, A. P. Fedotov, and B. K. Shembel, *Linear Ion Accelerators [in Russian]*, Gosatomizdat, 1962.

9. D. R. Pirs, *Theory and Design of Electron Beams [in Russian]* Izd. Sovetskoe radio, 1956.

10. M. D. Gabovich, "The effect of a space charge on the propagation of intense beams of charged par-

25 October 1966

Chelyabinsk